Write an algebraic expression to approximate the area under $f(x) = \ln x$ over the interval $[3, 10]$ using a left hand sum and n subintervals (as shown in class). **Do NOT evaluate the expression. Your answer must not use $f(\,)$ notation.**

SCORE: ______ / 3 PTS

$$\sum_{i=0}^{n-1} f(a+i\Delta x)\Delta x \qquad a=3 \qquad \Delta x = \frac{10-3}{n} = \frac{7}{n}$$

$$= \sum_{i=0}^{n-1} f\left(3+\frac{7i}{n}\right)\frac{7}{n}$$

1 1 1

$$= \sum_{i=0}^{n-1} \frac{7}{n} \ln\left(3+\frac{7i}{n}\right)$$

OR

$$\ln(3)\cdot\frac{7}{n} + \ln\left(3+\frac{7}{n}\right)\cdot\frac{7}{n} + \ln\left(3+\frac{14}{n}\right)\cdot\frac{7}{n} + \cdots + \ln\left(3+\frac{7(n-1)}{n}\right)\cdot\frac{7}{n}$$

Find $\lim_{x \to 0^-} \operatorname{csch} x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.** SCORE: ______ / 2 PTS

$$\lim_{x \to 0^-} \frac{2}{e^x - e^{-x}} = \underbrace{-\infty}$$ 1

1

AS $x \to 0^-$, $e^x \to 1^-$ AND $e^{-x} \to 1^+$, SO $\underbrace{e^x - e^{-x} \to 0^-}$,

$\frac{2}{0^-} \to -\infty$

If $\coth x = -3$, find $\operatorname{sech} x$ and $\sinh x$. **SCORE: ______ / 5 PTS**

$$\tanh x = \frac{1}{\coth x} = -\frac{1}{3} \quad 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$1 - \left(-\frac{1}{3}\right)^2 = \operatorname{sech}^2 x \quad 1$$

$$\operatorname{sech}^2 x = \frac{8}{9}$$

$$\operatorname{sech} x = \frac{2\sqrt{2}}{3} \quad 1$$

SINCE $\operatorname{sech} x > 0$
FOR ALL x

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{3}{2\sqrt{2}}$$

$$1 \quad = \frac{3\sqrt{2}}{4}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

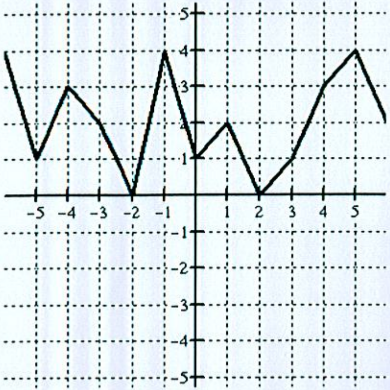
$$\sinh x = \tanh x \cosh x$$

$$= -\frac{1}{3} \cdot \frac{3\sqrt{2}}{4}$$

$$= -\frac{\sqrt{2}}{4} \quad 1$$

Estimate the area under the function shown on the right over the interval $[-5, 3]$ using the left hand sum with 4 equal width subintervals.

SCORE: ______ / 3 PTS



$\Delta x = \frac{3 - -5}{4} = 2$ $\frac{1}{2}$

INTERVALS = $[-5,-3]\ [-3,-1]\ [-1,1]\ [1,3]$

AREA $\approx f(-5)\cdot 2 + f(-3)\cdot 2 + f(-1)\cdot 2 + f(1)\cdot 2$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$= 1\cdot 2 + 2\cdot 2 + 4\cdot 2 + 2\cdot 2$ $\frac{1}{2}$

$= 18$ $\frac{1}{2}$

Prove the derivative of $\tanh^{-1} x$ without using the logarithmic definition of $\tanh^{-1} x$. **SCORE: ______ / 4 PTS**

You may use the derivatives of the non-inverse hyperbolic functions that were listed in your textbook without proving them.

NOTE: You must prove the Pythagorean–like identity for $\tanh x$ if you wish to use it.

$$y = \tanh^{-1} x$$

$$x = \tanh y \quad \frac{1}{2}$$

$$1 = \operatorname{sech}^2 y \frac{dy}{dx} \quad 1$$

$$\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y} \quad \frac{1}{2}$$

$$= \frac{1}{1-\tanh^2 y} = \frac{1}{1-x^2} \quad \frac{1}{2}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\frac{\cosh^2 y - \sinh^2 y}{\cosh^2 y} = \frac{1}{\cosh^2 y} \quad 1$$

$$1 - \tanh^2 y = \operatorname{sech}^2 y \quad \frac{1}{2}$$

Find $\frac{d}{dx}\tanh^{-1}(\operatorname{sech} x)$.

SCORE: ______ / 3 PTS

You may use the hyperbolic identities and derivatives from your textbook without proving them.

$$1 \quad \boxed{\frac{1}{1-\operatorname{sech}^2 x}} \cdot \underline{-\operatorname{sech} x \tanh x} \quad 1$$

$$= \frac{1}{\tanh^2 x} \cdot -\operatorname{sech} x \cancel{\tanh x}$$

$$= \boxed{\frac{-\operatorname{sech} x}{\tanh x}} \ \tfrac{1}{2} = \frac{-\operatorname{sech} x}{\tanh x} \cdot \frac{\cosh x}{\cosh x} = -\frac{1}{\sinh x} = \underline{-\operatorname{csch} x} \ \tfrac{1}{2}$$